Outline

- Singly Linked Lists
- Doubly Linked Lists
- Recursions

Data Structures and Algorithms

- Data structures
  - A data structure is a way of storing data in a computer so that it can be used efficiently.
- Algorithms
  - An algorithm is a finite sequence of well-defined instructions for solving a problem and guaranteed to terminate in a finite time.
  - An algorithm does not necessarily need to be executable in a computer, but can be converted into a program.

Algorithms versus Heuristics

- Heuristics
  - A heuristic is a finite sequence of well-defined instructions for partially solving a hard problem and guaranteed to terminate in a finite time.
  - A heuristic is not always guaranteed to find a solution while an algorithm is.
- Descriptions of algorithms
  - We will use Java-like pseudo code mixed with English to describe algorithms.

Abstract Data Types

- An Abstract Data Type (ADT) is a specification of a set of data and the set of operations that can be performed on the data.
  - Such a data type is abstract in the sense that it is independent of various concrete implementations.
  - The definition can be mathematical, or it can be programmed as an interface.
  - We need to implement an ADT by using a class to make it usable.

Linked Lists
**Singly Linked List**

- A singly linked list is a concrete data structure consisting of a sequence of nodes.
- Each node stores:
  - element
  - link to the next node

---

**The Node Class for List Nodes**

```java
/** Node of a singly linked list of strings. */
public class Node {
    private String element; // we assume elements are character strings
    private Node next; // Creates a node with the given element and next node.
    public Node(String s, Node n) {
        element = s;
        next = n;
    }
    /** Returns the element of this node. */
    public String getElement() {
        return element;
    }
    /** Returns the next node of this node. */
    public Node getNext() {
        return next;
    }
    // Modifier methods:
    /** Sets the element of this node. */
    public void setElement(String newElem) { element = newElem; }
    /** Sets the next node of this node. */
    public void setNext(Node newNext) { next = newNext; }
}
```

---

**The Class for a Singly Linked List**

```java
/** Singly linked list. */
public class SLinkedList {
    protected Node head; // head node of the list
    protected long size; // number of nodes in the list
    /** Default constructor that creates an empty list */
    public SLinkedList() {
        head = null;
        size = 0;
    }
    // ... update and search methods would go here ... 
}
```

---

**Inserting at the Head (1/2)**

1. Allocate a new node
2. Insert the new element
3. Have the new node point to old head
4. Update head to point to the new node

---

**Algorithm addFirst(v)**

```java
{ 
v.setNext(head); // make v point to the old head node
head = v; // make head point to the new node
size = size+1; // increment the node count
}
```

---

**Inserting at the Head (2/2)**

---

**Removing at the Head (1/2)**

1. Update head to point to next node in the list
2. Allow garbage collector to reclaim the former first node
Removing at the Head (2/2)

Algorithm removeFirst()
{
    if (head == null)
        Indicate an error: the list is empty;
    t = head;
    /* make head point to the next node (or null) */
    head = head.getNext();
    LastNext(null); //null out the next pointer of the removed node
    size = size - 1; //decrement the node count
}

Inserting at the Tail (1/2)

1. Allocate a new node
2. Insert the new element
3. Have the new node point to null
4. Have the old last node point to new node
5. Update tail to point to new node

Removing at the Tail (2/2)

Algorithm removeLast()
{
    if (size == 0)
        Indicate an error: the list is empty;
    else
    {  t = head;
        /* make t point to the head */
        while (t.getNext != null) /* find the last node */
        {  s = t;
            t = t.getNext(); }
        s.setNext(null); //null out the next pointer of the last node
        size = size - 1; //decrement the node count
    }
}

Inserting at the Tail (2/2)

Algorithm addLast(v)
{
    v.setNext(null); //make the new node v point to null object
    tail.setNext(v); //make the old tail node point to the new node
    tail = v; //make tail point to the new node
    size = size +1; //increment the node count
}

Removing at the Tail (1/2)

- Removing the tail node of a singly linked list cannot be done in constant time.
- We need to traverse the whole linked list to remove the tail node, which takes O(n) time, where n is the number of nodes in the linked list.

Doubly Linked List

- A doubly linked list is a concrete data structure consisting of a sequence of nodes.
- Each node stores:
  - element
  - link to the previous node
  - link to the next node
- Special trailer and header nodes

nodes/positions

elements

prev

next

elem
elem
elem
elem
elem

node
t

header

elem
elem
elem
elem
elem
elem

trailer

head

elem
elem
elem
elem
elem
elem

nodes/positions

elements
The Node Class of Doubly Linked List

```java
/** Node of a doubly linked list of strings */
public class DNode {
    protected String element; // String element stored by a node
    protected DNode next, prev; // Pointers to next and previous nodes
    /** Constructor that creates a node with given fields */
    public DNode(String e, DNode p, DNode n) {
        element = e;
        prev = p;
        next = n; // prev field names used to simplify implementation
    }
    /** Returns the element of this node */
    public String getElement() {
        return element;
    }
    /** Returns the previous node of this node */
    public DNode getPrev() {
        return prev;
    }
    /** Returns the next node of this node */
    public DNode getNext() {
        return next;
    }
    /** Sets the element of this node */
    public void setElement(String newElem) {
        element = newElem;
    }
    /** Sets the previous node of this node */
    public void setPrev(DNode newPrev) {
        prev = newPrev;
    }
    /** Sets the next node of this node */
    public void setNext(DNode newNext) {
        next = newNext;
    }
}
```

Inserting at the Head (1/2)

(a) Before the insertion

```
header -> Seattle -> New York
```

(b) After the insertion

```
header -> Sydney -> Rome -> Seattle -> New York
```

Inserting at the Head (2/2)

```
Algorithm addFirst(v)
{
    w = header.getNext(); // make w point to the first node
    v.setNext(w); // make v point to the old first node
    v.setPrev(header); // make v point to the header
    w.setPrev(v); // make the old first node point to v
    header.setNext(v); // make header point to v
    size = size+1; // increment the node count
}
```

Inserting in the Middle (1/2)

(a) Before the insertion

```
header -> Seattle -> New York
```

(b) After the insertion

```
header -> Sydney -> Rome -> Seattle -> New York
```

Inserting in the Middle (2/2)

```
Algorithm addAfter(v,z)
{
    w = v.getNext(); // make w point to the node after v
    z.setPrev(v); // link z to its predecessor, v
    z.setNext(w); // link z to its successor, w
    w.setPrev(z); // link w to its new predecessor, z
    v.setNext(z); // link v to its new successor, z
    size = size+1; // increment the node count
}
```

Removing at the Tail (1/2)

(a) Before the deletion

```
header -> Rome -> Seattle -> Sydney
```

(b) After the deletion

```
header -> Rome -> Seattle
```
Removing at the Tail (2/2)

Algorithm removeLast():
{
    if (size == 0)
        Indicate an error: this list is empty;
    v = trailer.getPrev(); // make v point to the last node
    u = v.getPrev(); // u points to the node before the last node
    trailer.setPrev(u);
    u.setNext(trailer);
    v.setPrev(null);
    v.setNext(null);
    size = size - 1;
}

Removing in the Middle (1/2)

Seattle
Sydney
New York
Rome

(a) Before the deletion

Programming with Recursion

Content of a Recursive Method

- **Base case(s)**
  - Values of the input variables for which we perform no recursive calls are called base cases (there should be at least one base case).
  - Every possible chain of recursive calls must eventually reach a base case.
- **Recursive calls**
  - Calls to the current method.
  - Each recursive call should be defined so that it makes progress towards a base case.

The Recursion Pattern

- **Recursion**: when a method calls itself.
- **Classic example—the factorial function**:
  - \( n! = 1 \cdot 2 \cdot 3 \cdots (n-1) \cdot n \)
- **Recursive definition**:

<pre>
  \[
  f(n) = \begin{cases} 
  1 & \text{if } n = 0 \\
  n \cdot f(n-1) & \text{else} 
  \end{cases}
  \]
</pre>

- As a Java method:
  // recursive factorial function
  public static int recursiveFactorial(int n) {
      if (n == 0) return 1; // basis case
      else return n * recursiveFactorial(n-1); // recursive case
  }

The Recursion Pattern

- **Recursion**: when a method calls itself.
- **Classic example—the factorial function**:
  - \( n! = 1 \cdot 2 \cdot 3 \cdots (n-1) \cdot n \)
- **Recursive definition**:

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  f(n) = \begin{cases} 
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  \]
</pre>

- As a Java method:
  // recursive factorial function
  public static int recursiveFactorial(int n) {
      if (n == 0) return 1; // basis case
      else return n * recursiveFactorial(n-1); // recursive case
  }

Removing in the Middle (2/2)

Algorithm remove(v):
{
    u = v.getPrev(); // make u point to the node before v
    w = v.getNext(); // w points to the node after v
    w.setPrev(u); // link out v
    u.setNext(w); // null out the pointers of v
    v.setPrev(null); // null out the pointers of v
    v.setNext(null); // decrement the node count
    size = size - 1; // decrement the node count
}
Visualizing Recursion

- Recursion trace
- A box for each recursive call
- An arrow from each caller to callee
- An arrow from each callee to caller showing return value

Example – English Rulers

- Define a recursive way to print the ticks and numbers like an English ruler:
  
  ```
  >>>
  0 1 2
  >>>
  3 4 5
  >>>
  6
  >>>
  ```

A Recursive Method for Drawing Ticks on an English Ruler

```java
public static void drawDiagonal(int n) {
    if (n > 0) {
        System.out.print(n);
        drawDiagonal(n - 1);
    }
}
```

Visualizing the DrawTicks Method

- An interval with a central tick length $L \geq 1$ is composed of the following:
  - an interval with a central tick length $L-1$,
  - a single tick of length 4,
  - an interval with a central tick length $L-1$.

Recall the Recursion Pattern

- Recursion: when a method calls itself
- Classic example—the factorial function:
  
  ```
  n! = 1 \cdot 2 \cdot 3 \cdots (n-1) \cdot n
  ```

- Recursive definition:
  
  ```
  f(n) = \begin{cases} 
  1 & \text{if } n = 0 \\
  n \cdot f(n-1) & \text{else}
  \end{cases}
  ```

- As a Java method:
  
  ```java
  public static int recursiveFactorial(int n) {
    if (n == 0) return 1; // basis case
    else return n * recursiveFactorial(n-1); // recursive case
  }
  ```
Linear Recursion

- Test for base cases.
  - Begin by testing for a set of base cases (there should be at least one).
  - Every possible chain of recursive calls must eventually reach a base case, and the handling of each base case should not use recursion.
- Recurse once.
  - Perform a single recursive call. (This recursive step may involve a test that decides which of several possible recursive calls to make, but it should ultimately choose to make just one of these calls each time we perform this step.)
  - Define each possible recursive call so that it makes progress towards a base case.

Reversing an Array

**Algorithm ReverseArray(A, i, j)**

```java
{  
  Input: An array A and nonnegative integer indices i and j
  Output: The reversal of the elements in A starting at index i and ending at j
  if (i < j)
  {
    swap A[i] and A[j];
    ReverseArray(A, i + 1, j - 1);
  }
}
```

Computing Powers

- The power function, \( p(x,n) = x^n \), can be defined recursively:
  \[
  p(x,n) = \begin{cases} 
  1 & \text{if } n = 0 \\
  x \cdot p(x,n-1) & \text{else}
  \end{cases}
  \]
- This leads to an power function that runs in O(n) time (for we make n recursive calls).
- We can do better than this, however.

Recursive Squaring

- We can derive a more efficient linearly recursive algorithm by using repeated squaring:
  \[
  p(x,n) = \begin{cases} 
  1 & \text{if } x = 0 \\
  x \cdot p(x,(n-1)/2)^2 & \text{if } n \text{ is odd} \\
  p(x,n/2)^2 & \text{if } n \text{ is even}
  \end{cases}
  \]
- For example,
  \[
  2^3 = 2 \cdot (2^2)^2 = 2^2 = 4^2 = 16
  
  2^5 = 2 \cdot 2^4 = 2 \cdot (2^2)^2 = 2(4)^2 = 32
  
  2^9 = 2 \cdot 2^8 = 2 \cdot (2^4)^2 = 2 \cdot (16)^2 = 64
  
  2^7 = 2 \cdot 2^6 = 2 \cdot (2^3)^2 = 2(8)^2 = 128.
  \]
A Recursive Squaring Method

Algorithm Power(x, n)
{
    Input: A number x and integer n = 0
    Output: The value x^n
    if (n = 0) return 1;
    if (n is odd )
        { y = Power(x, (n - 1)/ 2);
        return x*y*y; }
    else
        { y = Power(x, n/ 2);
        return y*y; }
}

Analyzing the Recursive Squaring Method

Algorithm Power(x, n)
{
    Input: A number x and integer n = 0
    Output: The value x^n
    if (n = 0) return 1;
    if (n is odd )
        { y = Power(x, (n - 1)/ 2);
        return x*y*y; }
    else
        { y = Power(x, n/ 2);
        return y*y; }
}

It is important that we used a variable twice here rather than calling the method twice.

Each time we make a recursive call we halve the value of n; hence, we make log n recursive calls. That is, this method runs in O(log n) time.

Tail Recursion

Tail recursion occurs when a linearly recursive method makes its recursive call as its last step.
The array reversal method is an example.
Such methods can be easily converted to non-recursive methods (which saves on some resources).
Example:
Algorithm IterativeReverseArray(A, i, j)
{
    Input: An array A and nonnegative integer indices i and j
    Output: The reversal of the elements in A starting at index i and ending at j
    while (i < j)
    { swap A[i] and A[j];
        i = i + 1;
        j = j – 1;
    }
}

Binary Recursion

Binary recursion occurs whenever there are two recursive calls for each non-base case.
Example: the DrawTicks method for drawing ticks on an English ruler.

Note the two recursive calls

A Binary Recursive Method for Drawing Ticks

Problem: add all the numbers in an integer array A:
Algorithm BinarySum(A, i, n)
{ Input: An array A and integers i and n
    Output: The sum of the n integers in A starting at index i
    if (n = 1) return A[i];
    return BinarySum(A, i, n/ 2) + BinarySum(A, i + n/ 2, n/ 2);
}

Example trace:
Computing Fibonacci Numbers

- Fibonacci numbers are defined recursively:
  \[ F_0 = 0 \]
  \[ F_1 = 1 \]
  \[ F_i = F_{i-1} + F_{i-2} \quad \text{for } i > 1. \]
- As a recursive algorithm (first attempt):

```java
Algorithm BinaryFib(k)
{
    Input: Nonnegative integer k
    Output: The kth Fibonacci number F_k
    if (k = 1) return k;
    else return BinaryFib(k - 1) + BinaryFib(k - 2);
}
```

Analyzing the Binary Recursion Fibonacci Algorithm

- Let \( n_k \) denote number of recursive calls made by \( \text{BinaryFib}(k) \). Then
  \[ n_0 = 1 \]
  \[ n_1 = 1 \]
  \[ n_2 = n_1 + n_0 + 1 = 1 + 1 + 1 = 3 \]
  \[ n_3 = n_2 + n_1 + 1 = 3 + 1 + 1 = 5 \]
  \[ n_4 = n_3 + n_2 + 1 = 5 + 5 + 1 = 11 \]
  \[ n_5 = n_4 + n_3 + 1 = 11 + 5 + 1 = 17 \]
  \[ n_6 = n_5 + n_4 + 1 = 17 + 11 + 1 = 29 \]
  \[ n_7 = n_6 + n_5 + 1 = 29 + 17 + 1 = 47 \]
  \[ n_8 = n_7 + n_6 + 1 = 47 + 29 + 1 = 77 \].

- Note that the value at least doubles for every other value of \( n_k \). That is, \( n_k > 2^{k/2} \). It is exponential!

A Better Fibonacci Algorithm

- Use linear recursion instead:

```java
Algorithm LinearFibonacci(k):
{
    Input: A nonnegative integer k
    Output: Pair of Fibonacci numbers (F_k, F_{k-1})
    if (k = 1) return (k, 0);
    else {
        (i, j) = LinearFibonacci(k - 1);
        return (i + j, i);
    }
}
```

- Runs in \( O(k) \) time.

Multiple Recursion

- Motivating example: summation puzzles
  - pot + pan = bib
  - dog + cat = pig
  - boy + girl = baby

- Multiple recursion: makes potentially many recursive calls (not just one or two).

Algorithm for Multiple Recursion

```java
Algorithm PuzzleSolve(k, S, U)
{
    Input: An integer k, sequence S, and set U (the universe of elements to test)
    Output: An enumeration of all k-length extensions to S using elements in U without repetitions
    for all e in U do
        Remove e from U;
        Add e to the end of S;
        if (k = 1)
            Test whether S is a configuration that solves the puzzle;
            if (S solves the puzzle) return "Solution found: " S;
        else
            PuzzleSolve(k - 1, S, U);
        Add e back to U;
        Remove e from the end of S;
}
```

Visualizing PuzzleSolve
References

1. Chapter 3, Data Structures and Algorithms by Goodrich and Tamassia.